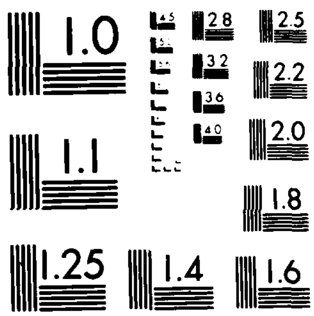
AD-A134 600 NONDESTRUCTIVE MEASUREMENT OF BULK RESIDUAL STRESSES 1/1
(U) HOUSTON UNIV TEX DEPT OF MECHANICAL ENGINEERING
K SALAMA OCT 83 N00014-82-K-0496

UNCLASSIFIED F/G 20/11 NL

END
DATE
FILMED
11 83
DTIC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS 1963 A

A134 600

DTIC FILE COPY

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER	2. GOVT ACCESSION NO. / 3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) NONDESTRUCTIVE MEASUREMENT OF BULK RESIDUAL STRESSES	5. TYPE OF REPORT & PERIOD COVERED Annual (July 82 - July 83)
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Kamel Salama	8. CONTRACT OR GRANT NUMBER(s) ONR N00014-82-K-0496
9. PERFORMING ORGANIZATION NAME AND ADDRESS Mechanical Engineering Department University of Houston Houston, Texas 77004	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS DD - 1473
11. CONTROLLING OFFICE NAME AND ADDRESS Office of Naval Research Arlington, VA 22217	12. REPORT DATE October 1983
	13. NUMBER OF PAGES 45
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) ONR Resident Representative Room 582, Federal Bldg. Austin, Texas 78701	15. SECURITY CLASS. (of this report) Non-Classified
	15a. DECLASSIFICATION DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Unlimited

Approved

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

Unlimited

DTIC ELECTE NOV 7 1983

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Residual Stress, Ultrasonics, Temperature Dependence of Ultrasonic Velocity

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

S/N 0102-014-6601

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

83 11 04 058

TABLE OF CONTENTS

A-1

NONDESTRUCTIVE MEASUREMENT OF BULK RESIDUAL STRESSES

ANNUAL REPORT
Contract N00014-82-K-0496

BACKGROUND

Residual stresses are those contained in a body which has no external traction or other sources of stress, such as thermal gradients or body forces. When the body is extrenally loaded, these stresses are called internal stresses, and, accordingly, residual stresses may be considered as a special case for vanishing external loads. Residual stresses result from non-uniform plastic deformation which includes cold working, forming, forging, heat treatment, etc. Their presence in manufactured components plays an important role in determining the behavior of the component when it is subjected to service loads and environment. It has also been shown that the residual stress distribution directly affects the growth rate and frequency of formation of stress induced cracks in steels.

Only in the case of surface stresses in components made of crystalline materials can nondestructive evaluation of stresses be performed by the X-ray diffraction method. Although considerably improved in the last ten years, this method still suffers from serious problems which severely restrict its applications. Ultrasonic methods appear to hold the best promise in measurements of bulk stresses in both crystalline and non-crystalline materials. Calculations have shown that ultrasonic velocity changes are linear functions of applied stress and combinations of second- and third-order elastic constants.

In the application of these calculations to determine unknown stresses, both the velocity in the absence of stress as well as third-order elastic constants have to be known independently. In addition, the measured velocity strongly depends on microstructural features which makes it necessary to develop a calibration between velocity and stress in order to be used in the determination of unknown stresses. Development of prefered orientations (texture) during deformation or fatigue, severely modify the third-order elastic constants. These problems can be solved when the differences between velocities of shear waves polarized perpendicular and parallel to stress direction are used. Due to these differences, a shift in phase will occur, and the out-of-phase components will interfere and cause a change in intensity. This method, however, does not have at present enough sensitivity, and requires an accurate determination of the shear velocity in the absence of stress.

Basically, the temperature dependences of the elastic constants are due to the anharmonic nature of the crystal lattice, and can be related to the pressure dependence of these constants. If we consider the isothermal bulk modulus B_T to be a function of pressure and temperature, it follows that

$$\left.\frac{\partial B_T}{\partial P}\right|_T = \frac{1}{\beta B_T}\left[\left.\frac{\partial B_T}{\partial T}\right|_V - \left.\frac{\partial B_T}{\partial T}\right|_p\right] \qquad (1)$$

where β is the volume coefficient of thermal expansion. Swenson has shown empirically that for many materials

$$\left.\frac{\partial B_T}{\partial T}\right|_V = 0, \qquad (2)$$

and it can be shown by differentiating $(B_S - B_T)$ that (1)

$$\left.\frac{\partial B_S}{\partial P}\right|_T = \left.\frac{\partial B_T}{\partial P}\right|_T \qquad (3)$$

to an accuracy of a few percent. Thus it follows that if Swenson's rule is correct,

$$\left.\frac{\partial B_S}{\partial P}\right|_T = \frac{1}{\beta BT} \cdot \left.\frac{\partial B_T}{\partial T}\right|_P \qquad (4)$$

to an accuracy of a few percent. The right-hand side can be calculated from the measurements of the temperature dependence of the second-order elastic constants. The left-hand side is calculated from the third-order elastic constants C_{ijk}, and eqn. (3) is then expressed as,

$$\frac{\partial B}{\partial T} = \frac{\beta}{9}[C_{111} + 3C_{112} + 3C_{113} + 2C_{123}] \qquad (5)$$

More rigorous relationships can be derived for the temperature dependences of longitudinal and the shear moduli which are related to the temperature dependence of the ultrasonic velocity v as,

$$\frac{\partial \ln M}{\partial T} = 2\frac{\partial \ln V}{\partial T} \qquad (6)$$

From this general argument, it can be seen that the temperature dependence of the ultrasonic velocity (longitudinal or shear) is a measure of the anharmonic effects generated when the solid is subjected to a stress. Changes in the anharmonic properties due to the presence of residual stresses can therefore be detected by the changes in the slope of the relationship between the ultrasonic velocity and the temperature. This slope can be determined with a high accuracy as its value depends only on measurements of the relative changes in the velocity and not on the absolute values. Experiments undertaken on aluminum, copper and A533B steel elastically deformed showed that the ultrasonic velocity, in the vicinity of room temperature, changed linearly with temperature, and the slope of the linear relationship changed considerably as the amount of applied stress was varied. In aluminum, copper and steel, the relative changes of the temperature dependence of longitudinal velocity increased respectively by 23% at a stress of 96 MPa, by 6% at a stress of 180 MPa, and by 15% at a stress of 240 MPa. The results also also showed that the temperature dependence of ultrasonic velocity $(dV/dT)_\sigma$ at applied stress σ may be represented by the relationship,

$$\frac{(dV/dT)_\sigma - (dV/dT)_o}{(dV/dT)_o} = K\sigma \qquad (7)$$

where $(dV/dT)_0$ is the temperature dependence at zero applied stress and K is a constant. The accuracy of the temperature dependence in measuring applied stress is estimated to be ±18 MPa in aluminum, ±25 MPa in copper and ±34 MPa in steel.

All the above studies have been performed when the stress is applied in a direction perpendicular to that of the ultrasonic wave propagation. In practice, the magnitude as well as the direction of applied or residual stress are not known. In order to apply the temperature dependence of ultrasonic velocity method to stress measurements, relationship similar to that expressed by eqn. (1) should be available for stress applied parallel as well as perpendicular to wave propagation. It is also important for the application of the method that the theoretical background for these relationships is established so that values of the constant K in eqn. (1) can be calculated using the solid anharmonic constants.

SUMMARY OF RESULTS AND DISCUSSION

The research on this contract was initiated in July 1982 in order to develop the method of the temperature dependence of ultrasonic velocity for the nondestructive evaluation of bulk residual stresses. Results previously obtained on aluminum, copper and steel have shown that this method offers a promising possibility for measurements of bulk stresses in these materials. The results also indicate that the method is less sensitive to variations caused by alloy composition and perhaps texture. Two objectives have been set for the research to be performed during the first year of this program. The first objective is to establish experimental relationships between the temperature dependence of ultrasonic velocity and stress applied in a direction parallel to and perpendicular to that of wave propagation. These relationships are to represent the variations of the temperature dependence when the stresses are applied in tension as well as in compression.

Appendix I includes two papers which describe the results obtained on the effects of applied stress on the temperature dependence of ultrasonic longitudinal velocity in aluminum 6061-T6. The first paper published in the Proceedings of the 1982 IEEE Ultrasonic Symposium, deals with the study performed to establish the relationship between ultrasonic velocity and compressive stress applied perpendicular to wave propagation. The results show that the temperature dependence decreases linearly by as much as 20% when a stress of 80 MPa is applied in this direction. The results also indicate that the relative changes in the temperature dependence in aluminum 6061-T6 are equal to those previously obtained on other three aluminum alloys,

and emphasize the insensitivity of the temperature dependence method to alloy composition.

The second paper in Appendix I which appears in the Proceedings of the 14th Symposium on NDE (1983), discusses the effects of stress applied in a direction parallel to and perpendicular to ultrasonic wave propagation on the temperature dependence of longitudinal velocity in aluminum 6061-T6. The results obtained in this study show that the temperature dependence of ultrasonic velocity increases linearly with either tensile or compressive stress when they are applied in a direction parallel to that of ultrasonic propagation. In the case of stress applied perpendicular to propagation, the results indicate that the temperature dependence decreases linearly with both stresses. The calibration curves constructed to relate the relative change in the temperature dependence to applied stress, give an estimate of ±10 MPa for the sensitivity of these curves in determining unknown applied stresses in aluminum used.

The second objective for the first year of this program was to establish the theoretical background for the relationship between the temperature dependence of ultrasonic velocity and stress. Experimental results obtained on three materials (aluminum, copper and steel) show that this relationship is linear and to a large extent does not depend on the alloying composition in each material. The experimental results also show that the slope of this relationship is large in aluminum and small in copper. Appendix II includes a manuscript under preparation, which describes the derivations of the temperature dependence of ultrasonic velocity with respect to stress, and the comparison between theory and experiment. The derivations are based on the expressions

obtained by Hiki, Thomas and Granato for the temperature dependences of isothermal elastic constants of single crystals using a quasiharmonic anisotropic continuum model. The temperature dependence of ultrasonic velocity in a polycrystal is derived in terms of second-, third- and fourth-order elastic constants using the Voight average of Young's modulus and rigidity modulus. This expression along with the experimental values of the temperature dependence of longitudinal velocity obtained in aluminum and copper, yield values of fourth-order elastic constants which are in good agreement with those computed by other investigators. Furthermore, the derivative of this expression with respect to stress is shown to be constant for constant values of the stress dependence of the third- and the fourth-order elastic constants. This result agrees with the experimental findings relating the temperature dependence of ultrasonic velocity to stress.

APPENDIX I

MEASUREMENT OF RESIDUAL STRESS USING THE TEMPERATURE DEPENDENCE OF ULTRASONIC VELOCITY

K. Salama, G. C. Barber and N. Chandrasekaran

Department of Mechanical Engineering
University of Houston
Houston, Texas 77004

Abstract

The temperature dependences of ultrasonic velocities are due to the anharmonic nature of the crystal lattice, and therefore can be used for stress measurements. Experiments performed on 6061-T6 aluminum show that, in the vicinity of room temperature, the ultrasonic longitudinal velocity decreases linearly with temperature, and the slope of the linear relationship varies considerably when the specimen is subjected to stress. For compressive stress applied perpendicular to wave propagation, the temperature dependence of the velocity is found to decrease linearly by as much as 20% at a stress of 80 MPa. The results also indicate that the relative changes in the temperature dependence of velocity as a function of stress are equal to those previously obtained on other aluminum alloys. This shows the insensitivity of the temperature dependence method to texture and alloy composition. The method thus offers a promising possibility for the nondestructive measurement of residual stress.

1. Introduction

There is a general agreement that ultrasonic methods appear to hold the best promise in the nondestructive measurements of bulk stresses in both crystalline and non-crystalline materials.[1,2] Calculations have shown that changes in ultrasonic velocities are linear functions of applied stress and unknown stresses can be determined when both the velocity in the absence of stress as well as third-order elastic constants are known independently. The measured velocity, however, strongly depends on microstructural features which makes it necessary to develop a calibration between velocity and stress for each material in order to be used in the determination of unknown stresses. In addition, development of preferred orientations (texture) during deformation or fatigue, severely modify the third-order elastic constants to be used for the calibration. These problems can be solved when the differences between velocities of shear waves polarized perpendicular and parallel to stress directions are used. Due to these differences, a shift in phase will occur, and the out-of-phase components will interfere and cause a change in intensity. This method, however, does not have at present enough sensitivity, and requires an accurate determination of the shear velocity in the absence of stress.

Basically, the temperature dependences of the elastic constants of a solid are due to the anharmonic nature of the crystal lattice.[3,4] A measure of the temperature dependence of the ultrasonic velocity can, therefore, be used to evaluate bulk stresses. Experiments undertaken on aluminum and copper[5,6] elastically deformed in compression showed that the ultrasonic velocity, in the vicinity of room temperature, changed linearly with temperature, and the slope of the linear relationship changed considerably as the amount of applied stress was varied. In aluminum, the relative changes of the temperature dependence of longitudinal velocity decreased linearly by as much as 23% at a stress of approximately 90 MPa. The results obtained on different types of aluminum alloys also indicate that the relative changes in the temperature dependence due to applied stress are insensitive to composition and texture, and the data obtained on these alloys can be represented by a single relationship.[7]

All the above studies have been performed when the stress is applied in a direction perpendicular to that of the ultrasonic wave propagation. In practice, the magnitude as well as the direction of applied or residual stress are not known. In order to apply the temperature dependence of ultrasonic velocity and stress should be available for stress applied parallel as well as perpendicular to wave propagation.

In this paper the effects of applied compressive stress on the absolute as well as the temperature dependence of the ultrasonic longitudinal velocity have been investigated. The experiments were performed with the stress applied in a direction parallel to and perpendicular to the ultrasonic wave propagation.

2. Experimental Procedure

The specimens used in the present study were made from one inch diameter bar stock of commercial 6061-T6 aluminum in the form shown in Fig. 1. All specimens were made identically except the overall length L was varied. The specimens were machined with a 2.54 cm diameter cap on each end which allowed the same specimen to be used for

0090-5607/82/0000-0877 $00.75 © 1982 IEEE

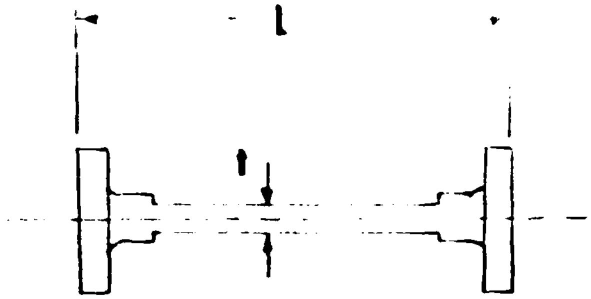


Fig. 1 Specimen used for ultrasonic velocity measurements.

stress applied in compression as well as in tension. The two caps at the ends were made flat and parallel to within ±0.002 cm in order to avoid diffraction effects in the ultrasonic beam during propagation. These caps were also connected to the center portion of the specimen by a 0.06 cm radius in order to minimize stress concentrations. After experiments of the stress applied parallel to the axis of specimen were completed, two parallel surfaces of thickness, t, were milled in the center of the specimen to allow measurements for stress applied perpendicular to wave propagation direction. In this case the ultrasonic waves were propagated in the center portion of the specimen where the stress is expected to be uniform.

The application of external stress was carried out with a model 1125 floor type Instron machine

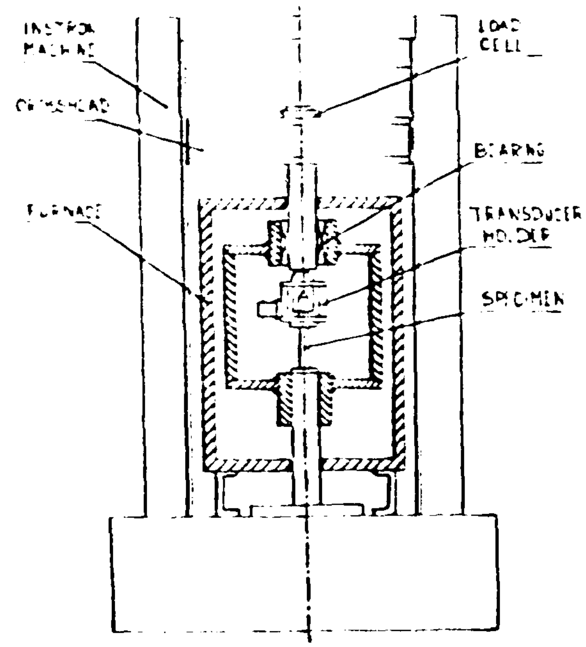


Fig. 2 Loading system used for the application of compressive stress in a direction parallel to the ultrasonic propagation.

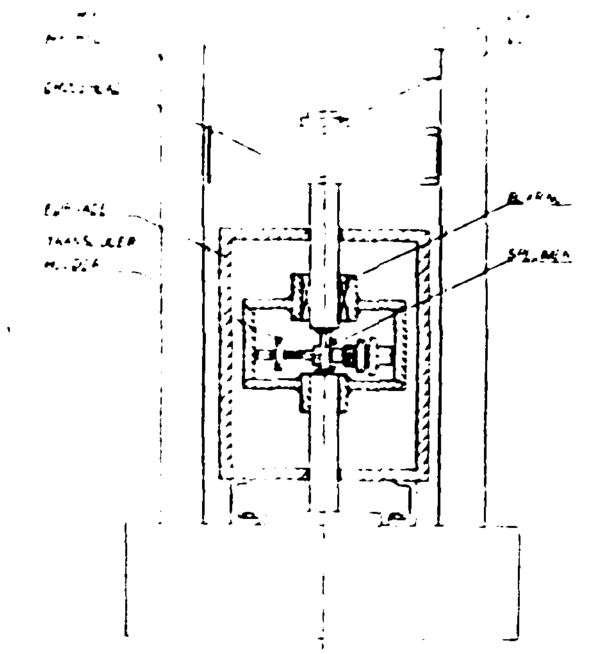


Fig. 3. Loading system used for the application of compressive stress in a direction perpendicular to the ultrasonic propagation.

The application of external stress was carried out with a model 1125 floor type Instron machine of 20,000 kg maximum load capacity. Two different types of loading arrangements are used in the present work. Figures 2 and 3 show the systems used for applying the compressive stresses. To help ensure the uniformity of stress in the specimens, special effort was made in designing these systems to minimize effects of misalignment between the axis of the specimen and the loading axis. A linear ball bearing served as the first alignment between the upper and lower loading axes and a hemispheric steel ball served as the second alignment between the specimen and the loading axis. The shafts used in the compression tests were made of surface hardened steel rod (R_c = 40) to resist possible wearing from the bearing during loading.

The ultrasonic velocity was measured using the pulse-echo overlap method which has been fully described elsewhere.[8] Figure 4 displays the experimental system used in this investigation. A pulse of approximately 1-μ sec duration of variable pulse-repetition rate is generated by the ultrasonic generator and impressed on a transducer of a fundamental frequency of 10 MHz which is acoustically bound to the specimen. The reflected rf echoes are received by the same transducer, amplified, and displayed on the screen of an oscilloscope. Two of the displayed echoes are then chosen and exactly overlapped by critically adjusting the frequency of the cw oscillator. This frequency f, accurately determined by the electronic counter, is employed

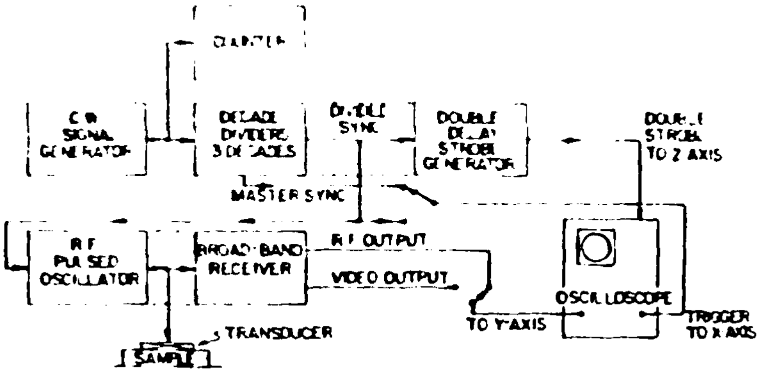


Fig. 4: Pulse-echo Overlap System.

to compute the ultrasonic velocity using the relation $V = 2lf$, where l is the length of the specimen. X-cut transducers are used for the generation of the longitudinal waves, which were used in all measurements.

The temperature control system is designed to enclose the specimen and its gripping assemblies in order to ensure stabilized temperature for the whole specimen during the time required for velocity measurements. These measurements were performed between 300 K and 370 K where the coupling condition between the transducer and the specimen was found to be satisfactory. At all temperatures, the actual temperature of the specimen was measured by a copper-constantan thermocouple attached directly to the specimen. The thermocouple along with a potentiometer provided a measurement of the temperature with an accuracy of ± 0.01 K.

3. Results

Temperature Dependence of Ultrasonic Velocity:

The effects of applied stress on the temperature dependence of ultrasonic longitudinal velocity were investigated on three specimens 1, 2 and 3 of 6061-T6 aluminum. Specimens 1 and 2 were used when the stress was applied parallel to wave propagation, while specimens 2 and 3 were used when the stress was applied perpendicular to the direction in which ultrasonic waves were propagated. Typical examples of the results obtained on specimen 1 showing the variations of ultrasonic velocity with temperature at the compressive stresses 0 and -19.8 MPa applied parallel to the propagation direction are shown in Fig. 5. From this figure it can be seen that, in the temperature range from 310-370 K, the ultrasonic velocity decreases linearly with temperature and the slope of the linear relationship varies when the stress is applied. A computer program was used to process the velocity-temperature data to determine the temperature dependence of ultrasonic velocity (dV/dT).

Table 1 lists the results of (dV/dT) obtained on the three specimens investigated when the stress was applied parallel to (specimens 1 and 2) as well as perpendicular to (specimens 2 and 3) the direction of wave propagation. Also included in this table are the offset uncertainties which measure the degree of linearity of the relationship between velocity and temperature. From these results one can see that the temperature dependence of ultrasonic velocity in aluminum increases or decreases according to whether the stress is applied parallel to or perpendicular to the direction in which the ultrasonic waves are propagated. The results also show that the changes in the temperature dependence are larger when the stress is perpendicular to wave propagation than when the stress is applied parallel to propagation direction.

Because the values of (dV/dT) at zero applied stress were found to vary among the specimens investigated, the relative change in the temperature dependence, L, due to the application of stress was calculated and its values are listed in column 6 of Table (1). The variations in the temperature dependence of the three specimens tested at zero stress are believed to be due to differences in the residual stress in these specimens. The values of L were calculated using the relationship,

$$L = \frac{(dV/dT)_o - (dV/dT)}{(dV/dT)} \qquad (1)$$

where (dV/dT) is the temperature dependence at zero applied stress.

The relative changes in the temperature dependence of ultrasonic velocity as a function of applied stress, obtained on the three specimens investigated are plotted in Fig. 6. The plots show that the data points obtained on specimens 1 and 2 when the stress was applied parallel to wave propagation, lie on a straight line which passes

Table (1) [illegible]

Specimen Number	Specimen Length cm	Applied Stress (MPa)	dV/dT (m/s·K)	Offset Uncertainty (m/s)	ΔS
		STRESS // PROPAGATION			
1	3.897	0	-0.969	.166	0.0
		-19.8	-0.982	.234	1.5
		-39.7	-0.996	.252	3.0
		-59.6	-1.016	.233	5.0
		-79.5	-1.032	.342	6.7
2	6.985	0	-1.009	.226	0.0
		-9.9	-0.975	.272	1.0
		-19.8	-0.984	.230	2.0
		-39.7	-0.998	.289	3.4
		-59.6	-1.022	.174	5.8
		STRESS ⊥ PROPAGATION			
2	t = 0.624	0.0	-1.007	.358	0.0
	L = 6.985	-16.6	-0.966	.326	-3.9
		-33.1	-0.933	.358	-7.1
		-66.2	-0.863	.197	-14.1
3	t = 0.624	0.0	-0.890	.332	0.0
	L = 7.638	-22.1	-0.845	.234	-5.3
		-44.1	-0.779	.313	-12.6
		-66.1	-0.736	.211	-17.5

through the origin with a slope equal to -8.63×10^{-4} per MPa and a correlation coefficient equal to -0.992. The figure also shows that the results obtained when the stress was perpendicular to wave propagation (specimens 2 and 3), also lie on a straight line which passes through the origin, but with a slope equal to 24.4×10^{-4} per MPa and a correlation coefficient of 0.982.

Stress Dependence of Ultrasonic Velocity:

The effects of applied stress on the velocity of longitudinal ultrasonic waves propagating parallel to and perpendicular to stress has been measured on specimens 1 and 3 respectively. The measurements were performed at room temperature with applied stress ranging from 0 to -59.6 MPa for specimen 1 and from 0 to -44.1 MPa for specimen 3. The results of these measurements are given in Table 2, which also lists the values of the relative change in the velocity $\Delta V/V$. Plots of these values as a function of applied stress for the two specimens are shown in Fig. 7. From this figure, one can see that $\Delta V/V$ increases or decreases linearly with stress according to whether the stress is applied parallel to or perpendicular to wave propagation. The slope of the straight line representing the velocity and stress when the ultrasonic waves are propagating parallel to stress is -0.210 m/s·MPa with a correlation coefficient equal to -0.998. The slope, however, is equal to 0.0832 m/s·MPa with a correlation coefficient equal to 0.993 when the stress is applied perpendicular to wave propagation.

... rs 1 and 3 of

Stress // Prop (Specimen 1)

Stress (MPa)	Velocity (m/s)	ΔV/V
0.0	6118.6	0.00
-9.9	6119.7	1.83
-19.9	6121.8	5.28
-29.8	6123.7	8.40
-39.7	6125.7	11.6
-49.7	6128.1	15.5
-59.6	6130.7	19.7

Stress Dependence dV/dσ = -0.210 m/s·MPa
Correlation Coefficient = -0.998

Stress ⊥ Prop (Specimen 3)

Stress (MPa)	Velocity (m/s)	$(\Delta V/V)10^4$
0.0	6191.2	0.00
-5.5	6191.1	0.18
-11.0	6190.6	-1.03
-16.5	6190.2	-1.70
-22.1	6189.5	-2.78
-27.6	6188.9	-3.76
-33.1	6188.2	-4.86
-38.6	6187.5	-6.06
-44.1	6186.9	-7.01

Stress Dependence dV/dσ = 0.0832 m/s·MPa
Correlation Coefficient = 0.993

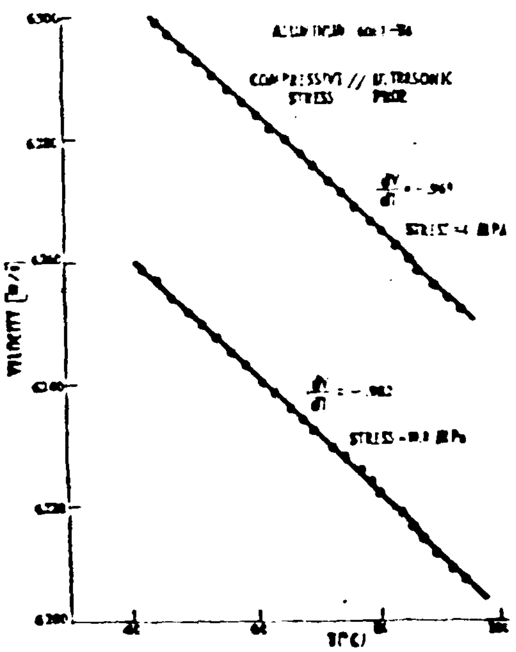


Fig. 5 Effect of applied compressive stress on the temperature dependence of ultrasonic longitudinal velocity in the aluminum alloy 6061-T6. The stress is applied in a direction parallel to the ultrasonic propagation.

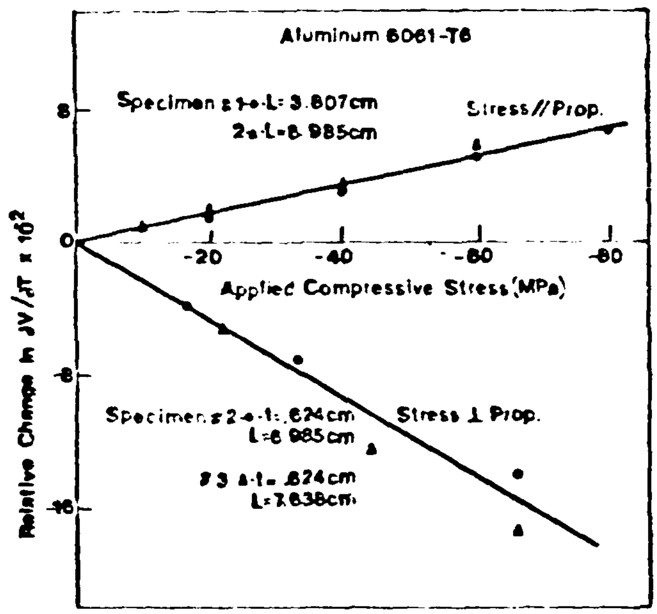


Fig. 6 Effect of applied compressive stress on the relative change in the temperature dependence of longitudinal ultrasonic velocity in 6061-T6 aluminum. Stress was applied parallel to and perpendicular to wave propagation.

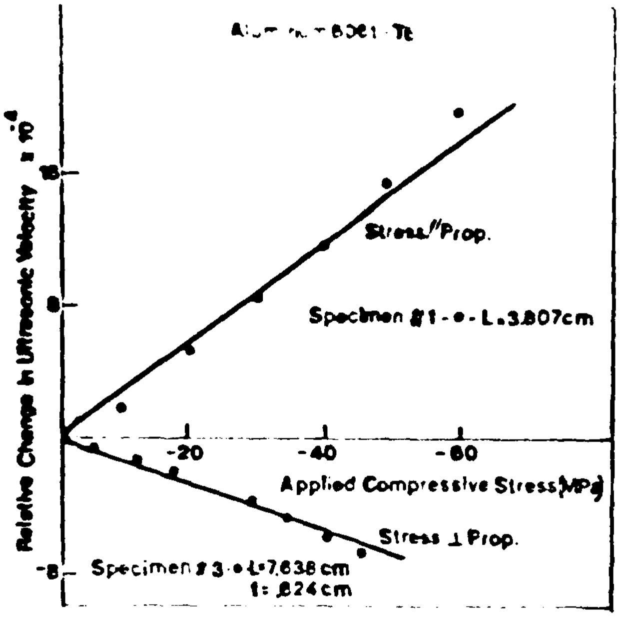


Fig. 7 Relative change in ultrasonic velocity as a function of applied compressive stress in 6061-T6 aluminum. Stress was applied parallel to and perpendicular to ultrasonic wave propagation.

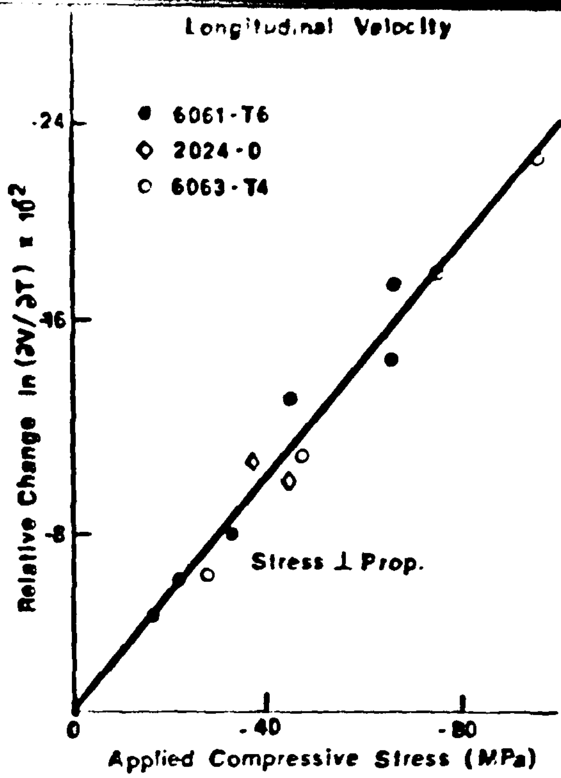


Fig. 8 Relative change in the temperature dependence of ultrasonic velocity as a function of applied stress in three aluminum alloys.

4. DISCUSSION

1. Temperature Dependence of Ultrasonic Velocity

The effects of compressive stresses on the temperature dependence of longitudinal ultrasonic velocity have been studied in the aluminum alloys 2024-0 and 6063-T4 by Salama and Ling[9]. These experiments were performed with the stress applied in a direction perpendicular to the ultrasonic propagation. The results obtained by these authors show that the relative change in the temperature dependence of ultrasonic velocity is a linear function of the applied elastic compressive stress and can be represented by the relationship

$$\ell = \frac{(dV/dT)_c - (dV/dT)}{(dV/dT)} = K\sigma \qquad (2)$$

where (dV/dT) is the temperature dependence at zero applied stress, $(dV/dT)_c$ is the temperature dependence at an applied stress σ, and K is a constant equal to 2.3×10^{-3} per MPa.

In the present investigation, the experiments were performed to study the effects of compressive stress on the temperature dependence of longitudinal ultrasonic velocity on three specimens (1,2 and 3) of the aluminum alloy 6061-T6. The relative changes in the temperature dependence of longitudinal ultrasonic velocity as a function of applied stress obtained in this work as well as those published in reference 9 are plotted in Fig. (8). The solid data points in this figure represent the data obtained in this investigation and the hollow data points represent those obtained in this reference. From this figure it can be seen that the relative changes in the temperature dependence of these three aluminum alloys as a function of compressive stress can be represented by Eq. (2) with K equal to 2.38×10^{-3} per MPa. This result emphasizes an earlier finding concerning the insensitivity of the temperature dependence of ultrasonic velocity to texture and alloy composition, and makes this method more suitable for the nondestructive evaluation of stress than direct velocity measurements. The latter method has been shown to be strongly influenced by metallurgical variables and requires calibration for each alloy or perhaps for each specimen[10].

On the other hand, the effect of stress on the temperature dependence of ultrasonic longitudinal velocity becomes much smaller and opposite in sign

when the stress is applied parallel to the direction in which the waves are propagated. This agrees with the results obtained by Chern and Heyman[11] who also observed small changes in the temperature dependence with stress in 2024-T4 aluminum when the stress was applied in the same direction of wave propagation. From Figs. 6 and 7 it is interesting to note that the behavior of the temperature dependence with stress (Fig. 6) is opposite to that of the velocity itself (Fig. 7). The comparison between the effects of stress on the temperature dependence and on the velocity will be discussed below.

2. Stress Dependence of Ultrasonic Velocity

For a longitudinal ultrasonic wave propagating parallel to the direction of applied stress, the change in velocity with stress, $dV/d\sigma$, may be expressed as[12]

$$\frac{dV}{d\sigma} = \frac{-1}{2\rho V(3\lambda + 2\mu)} \left[2l + \lambda + \frac{\lambda + \mu}{\mu}(4m + 4\lambda + 10\mu)\right] \tag{3}$$

In Eq. 3 λ and μ are the second order elastic constants, l and m are the third order elastic constants of Murnaghan[12], ρ is density, V is ultrasonic velocity, and σ is applied stress. This equation indicates that for small stresses, $dV/d\sigma$ is a constant. Heyman and Chern[13] have used the following relationship to measure axial stresses in fasteners

$$\frac{\Delta f}{f} = \left[\frac{1}{V}\frac{dV}{d\sigma} - \frac{1}{E}\right]\Delta\sigma \tag{4}$$

where f is the round trip frequency of a longitudinal wave propagating in the axial direction of the fastener, V is the ultrasonic velocity, E is Young's modulus and σ is applied stress. For small strains, the term in brackets is constant and Eq. (4) can be written as

$$\frac{\Delta f}{f} = H^{-1} = \Delta\sigma \tag{5}$$

where

$$H^{-1} = \left[\frac{1}{V}\frac{dV}{d\sigma} - \frac{1}{E}\right]^{-1} \tag{6}$$

For aluminum, they found H^{-1} to be a constant equal to -1.86×10^4 MPa.

In the present investigation, the effects of compressive applied stress on ultrasonic longitudinal velocity are shown in Fig. 7. From this figure, it can be seen that for stress applied in a direction parallel to that of ultrasonic propagation, the results give a value of -0.210 m/s·MPa for the stress dependence, $dV/d\sigma$. Substituting the values $V = 6200$ m/s, $E = 70.3$ GPa, and $dV/d\sigma = -0.210$ m/s·MPa into Eq. (6) yields $H^{-1} = -1.96 \times 10^4$ MPa. This value is in excellent agreement with the value of H^{-1} equal to -1.86×10^4 MPa obtained by Heyman and Chern for 2024-T6 aluminum.

For stress applied perpendicular to wave propagation, the change in velocity with stress is given by[12]

$$\frac{dV}{d\sigma} = \frac{-1}{2\rho V(3\lambda + 2\mu)} \left[2l - \frac{2\lambda}{\mu}(m + \lambda + 2\mu)\right] \tag{7}$$

Table (3) - Summary of important quantities found in this investigation for aluminum 6061-T6.

Row #	Quantity	Stress ⊥ Propagation	Stress // Propagation
1)	$\left[\frac{\partial V}{\partial T}\right]$; $\sigma = 0$	$-0.985 \frac{m/s}{K}$	$-0.985 \frac{m/s}{K}$
2)	$\dfrac{\left[\frac{\partial V}{\partial T}\right]_\sigma - \left[\frac{\partial V}{\partial T}\right]}{\sigma}$	$-23.4 \times 10^4 \frac{m/s}{K \cdot MPa}$	$8.50 \times 10^{-4} \frac{m/s}{K \cdot MPa}$
3)	$\left[\frac{\partial T}{\partial \sigma}\right]_T$; Room Temperature	$0.0832 \frac{m/s}{MPa}$	$-0.210 \frac{m/s}{MPa}$

where all the terms in the equation are as defined above. Since the values of the third order elastic constants l and m of aluminum are both negative and of the same magnitude[12], the value of $(dV/d\sigma)$ represented by eq. (7) should be negative and smaller than that of eq. (3). This agrees with the results shown in Fig. 7 where $(dV/d\sigma)$ in case of stress applied perpendicular to wave propagation is negative and has a smaller value (0.0632 m/s.MPa) than that obtained when the stress is applied parallel to the propagation direction (-0.210 m/s.MPa).

3. Relationship Between Temperature and Stress Dependences

Table (3) lists a summary of the important relationships obtained in this investigation for aluminum 6061-T6. The temperature dependence quantity given in row 1 represents the average value of the temperature dependence of ultrasonic velocity found in this investigation when the applied stress is equal to zero. The quantity in row 2 represents the change in the temperature dependence due to the application of compressive stress, which is calculated from Fig. 6. The values of the stress dependence at room temperature, are shown in row 3.

From Table (3), it can be seen that the product of the change in the temperature dependence due to applied stress (row 2) and the stress dependence at room temperature (row 3) is the same for both the parallel and the perpendicular configurations. The relationship between these two quantities can then be written as,

$$\left[\left(\frac{\partial V}{\partial T}\right)_c - \left(\frac{\partial V}{\partial T}\right)\right]\left[\left(\frac{\partial V}{\partial c}\right)\right] = k_1 c \qquad (8)$$

where

$$k_1 = -1.915 \pm 0.035\ [(m/s)/MPa]^2(1/°K).$$

This relationship indicates that the change in the temperature dependence with applied stress is inversely proportional to the dependence of the velocity on stress regardless of the relative direction of the stress with respect to the wave propagation. Equation 8 can thus be used to predict the magnitude as well as the sign of the change in the temperature dependence with stress when the corresponding dependence of velocity is known. This later can be calculated from available relationships similar to those expressed in eqs. 3 and 7.

5. Acknowledgement

This work is supported by the Office of Naval Research under Contract N00014-82-K-0496 and the Electric Power Research Institute under Contract T107-2.

6. References

1. Proceedings of a Workshop on Nondestructive Evaluation of Residual Stress, NTIAC-72-2 (1976).

2. James, M. R. and Buck, O., Critical Reviews in Solid State and Materials Sciences, p. 61 (1980).

3. Huntington, H. B., Solid State Phys. 7, 213 (1958).

4. Garber, J. A. and Granato, A. V., Phys. Rev. 131A, 3990 (1975).

5. Salama, K., Collins, A. L. W. and Wang, J. J., Proceedings DARPA/AF Review of Progress in Quantitative NDE, p. 265 (1980).

6. Salama, K. and Ling, C. K., Proceedings ARPA/AFML Review of Progress in Quantitative NDE, p. 96 (1979).

7. Salama, K., Ling, C. K. and Wang, J. J., Experimental Techniques 5, 14 (1981).

8. Papadakis, E. P., J. Acoust. Soc. Am. 42, 1045 (1967).

9. Salama, K. and Ling, C. K., J. Appl. Phys. 51, 1505 (1980).

10. Kino, G. S., Barnett, D. M., Grayeli, N., Hermann G., Hunter, J. B., Ilic, D. B., Johnson, G. C., King, R. B., Scott, M. P., Shyne, J. C. and Steele, C. R., Journal of Nondestructive Evaluation 1, 67 (1980).

11. Chern, E. J., Heyman, J. S. and Cantrell, J. H. Proceedings IEEE Ultrasonic Symposium p. 960 (1981).

12. Hughes, D. S. and Kelly, J. L., Phys. Rev. 92, 1145 (1953).

13. Heyman, J. S. and Chern, E. J., Proceedings ASTM Symposium on Ultrasonic Measurements of Stress, (1981).

14. Smith, R. T., Stern, R. and Stephens, R. W. B. J. Acoust. Soc. Am. 40, 1002 (1966).

14th Symposium on Nondestructive Evaluation

NONDESTRUCTIVE STRESS MEASUREMENTS IN ALUMINUM

K. Salama, G. C. Barber and N. Chandrasekaran
Department of Mechanical Engineering
University of Houston
Houston, Texas 77004

Abstract

The effects of applied elastic stress on the temperature dependence of 10 MHz ultrasonic longitudinal velocity have been studied in aluminum 6061-T6. Velocities of longitudinal ultrasonic waves were measured as a function of temperature in several specimens using a pulse-echo-overlap system. Experiments were performed with the stress applied in a direction parallel to and perpendicular to the ultrasonic propagation direction. In all temperature dependence measurements, the ultrasonic velocity is found to decrease linearly with temperature, and the slope of the line of best fit of ultrasonic velocity versus temperature is found to vary considerably when the specimens are subjected to stress. The results obtained when the stress is applied in a direction parallel to the ultrasonic propagation show that the temperature dependence of ultrasonic velocity increases linearly with either applied tensile or compressive stress. In the case of stress applied perpendicular to the ultrasonic propagation, the results indicate that the temperature dependence decreases linearly with either applied tensile or compressive stress. Calibration curves relating the relative change in the temperature dependence of ultrasonic velocity to applied stress are constructed. Using these calibration curves, the sensitivity in determining unknown applied stresses in aluminum is estimated to be ± 10 MPa.

1. INTRODUCTION

Materials in machine components are always in a state of stress. This stress can be applied stress due to external loading, residual stress within the material without any external loading, or a combination of applied and residual[1]. Applied stresses can be calculated using formulae from mechanics of materials or can be found experimentally using strain measuring devices attached to the machine. Residual stresses, however, can seldom be calculated because there is usually no data on which stress calculations are based. There are presently several methods of experimentally finding residual stresses by destructive means, such as hole drilling and ring coring[2]. To measure residual stresses in a body non-destructively, however, several approaches based on three major methods have been proposed. These methods are x-ray diffraction, electromagnetics, and ultrasonics. None of these methods measures stress directly. X-ray diffraction measures lattice strain, electromagnetic methods measure magnetic properties, and ultrasonic methods measure ultrasonic velocity. All of these methods have limitations which prevent any of them from being used in all stress measurement applications. For the nondestructive evaluation of bulk residual stresses in crystalline and non-crystalline materials, the methods using ultrasonic techniques seem to hold the best promise.

The temperature dependences of the elastic constants of a solid are due to the anharmonic nature of the crystal lattice.[3,4] A measure of the temperature dependence of the ultrasonic velocity can, therefore, be used to evaluate bulk stresses. Experiments undertaken on aluminum and copper[5,6] elastically deformed in compression showed that the ultrasonic velocity, in the vicinity of room temperature, changed linearly with temperature, and the slope of the linear relationship changed considerably as the amount of applied stress was varied. In aluminum, the relative changes of the temperature dependence of longitudinal velocity decreased linearly by as much as 23% at a stress of

[illegible] changes in the temperature dependences due to applied stress are insensitive to composition and texture, and the data obtained on these alloys can be represented by a single relationship, as

$$\Delta = \frac{(dV/dT)_{\sigma} - (dV/dT)}{(dV/dT)} = K\sigma \qquad (1)$$

where (dV/dT) is the temperature dependence at zero applied stress, $(dV/dT)_{\sigma}$ is the temperature dependence at an applied stress σ, and K is a constant equal to 2.3×10^{-5} per MPa. In addition, the effects of tensile elastic stresses on the temperature dependence of longitudinal velocity were studied using the aluminum alloys 2024-0, 3003-T251, and 2024-T351 by Salama and Wang (5). The results from these experiments were found to satisfy Eq. (1) with the constant K equal to -1.89×10^{-5} per MPa. All the above studies have been performed when the stress was applied in a direction perpendicular to that of the ultrasonic wave propagation.

In the present investigation, experiments were performed to study the effects of both compressive and tensile elastic stresses on the temperature dependence of longitudinal ultrasonic velocity in three specimens of the aluminum alloy 6061-T6. The experiments were performed with the stress applied in a direction parallel to and perpendicular to the ultrasonic wave propagation.

2. EXPERIMENTAL PROCEDURE

The specimens used in the present study were made from one inch diameter bar stock of commercial 6061-T6 aluminum in the form shown in Fig. 1. All specimens were made identical except the overall length l was varied. The specimens were machined with a 2.54 cm diameter cap on each end which allowed the same specimen to be used for stress applied in compression as well as in tension. The two caps at the ends were made flat and parallel to within ±0.002 cm in order to avoid diffraction effects in the ultrasonic beam during propagation. These caps were also connected to the center portion of the specimen by a 0.06 cm radius in order to minimize stress concentrations. After experiments of the stress applied parallel to the axis of specimen were completed, two parallel surfaces of thickness, t, were milled in the center of the specimen to allow measurements for stress applied perpendicular to wave propagation direction. In this case the ultrasonic waves were propagated in the center portion of the specimen where the stress is expected to be uniform.

The application of external stress was carried out with a model 1125 floor type Instron machine of 20,000 kg maximum load capacity. Four different types of loading arrangements [illegible] described in detail elsewhere [illegible] ensure the uniformity of stress in specimens, special effort was made in designing these stress application systems to minimize any effects of misalignment between the axis of the specimen and the loading axis. For tensile testing, this was achieved by using two universal joints between specimen and connections to load cell and to the loading frame. For compressive testing, a two-stepped alignment system was used. A linear ball bearing served as the first alignment between the upper and lower loading axes and the hemispheric steel ball served as the second alignment between the specimen and the loading axis.

The ultrasonic velocity was measured using the pulse-echo-overlap method which has been fully described elsewhere[6]. X-cut transducers are used for the generation of the longitudinal waves, which were used in all measurements. The transducer is placed on the specimen by means of special holders which are designed to clamp to the specimen as shown in Fig. 2 for the parallel configuration and Fig. 3 for the perpendicular configuration. The spring-supported plunger serves as the inner conductor of the coaxial cable which carries the 10 MHz electrical signals to and from the transducer which is bonded to the specimen. These signals are transmitted from the plunger by a piece of teflon-coated wire to a BNC receptacle which is mounted directly to the transducer holder. The spring is used to produce a pressure-coupled transducer which is required to insure uniformity of the thin layer bond. A small clamping force of 30 to 50 Newtons was used in the present work.

A temperature control system is designed to enclose the specimen gripping assemblies to ensure stabilized temperature for the entire specimen during the time required for velocity measurements. All experiments were performed between approximately 310°K and 370°K where the coupling condition between the transducer and the specimen was found to be satisfactory. A test furnace was used for heating. This furnace provides a uniform temperature performance by making use of a recirculating blower and planum system. The furnace is equipped with a temperature controller which is capable of providing precision temperature control. Before taking velocity measurements, the furnace is set at a desired maximum temperature. After this desired temperature is reached, the furnace is turned off and the cooling rate of the specimen is controlled to achieve a constant cooling rate. The temperature of the specimen is measured by a copper-constantan thermocouple attached directly to the specimen. The thermocouple along with a potentiometer provides the measurement of the specimen temperature to an accuracy ±0.1°K.

3. RESULTS

The velocity of longitudinal ultrasonic waves propagating perpendicular to the applied stress

[illegible] of the [illegible] temperature [illegible] changes of the [illegible] ultrasonic velocity obtained on specimen 2 at the applied tensile [illegible] MPa and 66.8 MPa are shown in Fig. 4. This figure shows that, in the temperature range from 310°K to 370°K, ultrasonic velocity decreases linearly with temperature. A computer program was used to determine the temperature dependence of ultrasonic velocity, dV/dT.

[illegible] results are given in Table [illegible] Table (1) are the relative [illegible] in the temperature dependence, due to application of stress. These relative percentage changes of the temperature dependence are plotted as a function of applied stress in Fig. (5). From this figure, it can be seen that the data points for tensile stress lie on a straight line which passes through the origin. This line has a slope of -1.18×10^{-3}/MPa and a correlation coefficient equal to -0.972. The data points for compressive stress also lie on a straight line which passes through the origin and has a slope of 2.44×10^{-3}/MPa and a correlation coefficient equal to 0.982.

Table (1) - Variations of the temperature dependence of longitudinal ultrasonic velocity with applied tensile and compressive stress in 6061-T6 aluminum. The stress is applied perpendicular to the direction of ultrasonic propagation.

Specimen Number	Specimen Dimensions cm (inch)	Applied Stress (MPa)	dV/dT (m/s·K)	Offset Uncertainty (m/s)	Δ%
1	t = 0.624 (0.2455)	66.2	-.9177	.158	-8.04
	l = 6.985 (2.750)	44.1	-.9518	.198	-4.62
		22.1	-.9530	.364	-4.50
		0.0	-1.0076	.358	0.00
		-16.6	-.9657	.326	-3.88
		-33.1	-.9355	.358	-7.11
		-66.1	-.8631	.197	-14.09
2	t = 0.495 (0.195)	66.8	-.9396	.224	-8.44
	l = 6.351 (2.5005)	46.7	-.9655	.234	-5.62
		26.7	-.9973	.192	-2.81
		0.0	-1.0230	.192	0.00

Table (2) - Values of slope, Y-intercept, and correlation coefficient of the lines of best fit of dV/dT versus applied stress in aluminum 6061-T6. The stress is applied perpendicular to the direction of ultrasonic propagation.

Specimen Number	Specimen Dimensions cm (inch)	Applied Stress	Slope (m/s·K MPa)	Correlation Coefficient	Y-Intercept (m/s·K)
1	t = 0.624 (0.2455) l = 6.985 (2.750)	Tensile	1.22×10^{-3}	.941	-0.9979
2	t = 0.495 (0.195) l = 6.351 (2.5005)	Tensile	1.26×10^{-3}	.995	-1.0262
1	t = 0.624 (0.2455) l = 6.985 (2.750)	Compressive	-2.15×10^{-3}	-.999	-1.0047

Table (1) lists the values of the temperature dependence of longitudinal velocity and the offset uncertainty in specimens 1 and 2 when they were subjected to applied stresses ranging from -66.2 MPa to 66.8 MPa. This table shows that the magnitude of the temperature dependence of ultrasonic velocity decreases linearly with applied compressive or tensile stress with the maximum value for the temperature dependence of ultrasonic velocity occuring at 0.0 MPa. The values of the slope, the intercept, and the correlation coefficient of the lines of best fit

The velocity of longitudinal ultrasonic waves propagating parallel to applied stress was measured as a funtion of temperature on the two aluminum specimens 2 and 3. Table (3) lists the results obtained on these two aluminum 6061-T6 specimens investigated in the parallel configuration. Included in this table

[illegible] compressive and tensile stresses ranging between -79.5 MPa and 59.6 MPa. Also listed in Table (3) are the offset uncertainties which measure the linearity of the relationship between velocity and temperature. The results of Table (3) are plotted in Fig. (6) which illustrates the effect of applied compressive and [illegible] of ultrasonic velocity increases linearly with either applied compressive or tensile stress, with the minimum value of the temperature dependence occurring at 0.0 MPa. The values of the slope, the intercept, and the correlation coefficient for the lines of best fit for the experimental results shown in Fig. (6) are given in Table (4). From Tables (2) and (4), it can be seen that the slopes of

Table (3) - Variations of the temperature dependence of longitudinal ultrasonic velocity with applied tensile and compressive stress in aluminum 6061-T6. Stress is applied parallel to the direction of ultrasonic propagation.

Specimen Number	Specimen Length cm (inch)	Applied Stress (MPa)	dV/dT (m/s·K)	Offset Uncertainty (m/s)	ΔV
2	6.351 (2.5005)	59.7	-1.0529	.423	1.37
		29.8	-1.0484	.185	.93
		19.8	-1.0469	.163	.79
		9.9	-1.0417	.248	.29
		0	-1.0146	.243	0
		-19.8	-1.0231	.236	.88
		-49.7	-1.0355	.266	2.10
		-59.6	-1.0427	.351	2.81
3	3.807 (1.499)	59.6	-1.0072	.466	3.14
		49.7	-.9960	.318	2.02
		29.8	-.9944	.530	1.84
		9.9	-.97[illegible]	.326	.35
		0	-.9692	.166	0
		-19.8	-.9[illegible]	.234	1.53
		-39.7	-.9961	.252	2.99
		-59.6	-1.0155	.233	5.02
		-79.5	-1.1317	.342	6.69

Table (4) - Values of slope, intercept, and correlation coefficient taken from lines of best fit of dV/dT versus applied stress in aluminum 6061-T6. The stress is applied parallel to the direction of ultrasonic propagation.

Specimen Number	Specimen Length cm (inch)	Applied Stress	Slope (m/s·K MPa)	Correlation Coefficient	Y-Intercept (m/s·K)
1	6.351 (2.5005)	Tensile	-3.53×10^{-4}	-.981	-1.0387
3	3.807 (1.499)	Tensile	-4.82×10^{-4}	-.948	-.9764
1	6.351 (2.5005)	Compressive	4.56×10^{-4}	.996	-1.0142
3	3.807 (1.499)	Compressive	7.99×10^{-4}	.997	-.9672

tensile stresses on the temperature dependence of ultrasonic velocity. This figure shows that, within the applied stress range used in these measurements, the magnitude of the the lines of best fit for the perpendicular configuration are much larger and opposite in sign to those obtained in the parallel configuration.

[illegible] with the [illegible] [illegible]. The intercept for the tensile portion of the graph is also [illegible] than that the intercept for the compressive portion of the graph. In the case of tensile loading, bending stresses are introduced and likely to exist throughout the entire length of the specimen. This non-uniform loading is expected to affect the absolute magnitude of dV/dT, but not the relative differences between quantities as indicated by the consistancy of the slopes of the lines of best fit of dV/dT versus stress in these specimens. In the case of compressive loading, the bottom end cap of the specimen is loaded uniformly which gives the compression loading a better probability of uniform stress throughout the ultrasonic path.

Because the values of the intercept of the lines of best fit were found to vary among the specimens investigated, the relative change in the temperature dependence, I, due to the application of stress was calculated and its values are listed in column 6 of Table (3). The values of I were calculated using the relationship,

$$I = \frac{(dV/dT)_\sigma - (dV/dT)_0}{(dV/dT)_0} \qquad (2)$$

where (dV/dT)[illegible] is the temperature dependence given by the intercept of the line of best fit and (dV/dT)[illegible] is the temperature dependence at an applied stress, σ. The variations in the temperature dependence of the three specimens tested at zero stress are believed to be due to differences in the residual stress in these specimens.

The relative changes in the temperature dependence, obtained on all three specimens investigated, are plotted in Fig. 7. The plot for tensile stress shows that the points lie on a straight line which passes through the origin with a slope equal to 4.39×10^{-4} per MPa and a correlation coefficient equal to 0.95[illegible]. The plots for compressive stress show that the points for specimen 3 lie on a straight line that passes through the origin with a slope equal to -8.63×10^{-4} per MPa and a correlation coefficient equal to -0.992. The points for specimen 2 for compressive stress also lie on a straight line that passes through the origin with a slope equal to -4.56×10^{-4} per MPa and a correlation coefficient equal to -0.996. There is no explanation for why the slope of the data obtained on specimen 2 is smaller than that found from the data obtained on specimens 3.

4. DISCUSSION

In the present investigation, experiments were performed to study the effects of both compressive and tensile elastic stresses on the temperature dependence of longitudinal ultrasonic [illegible] relative changes [illegible] longitudinal ultrasonic velocity as a function applied stress obtained in this work as well as those published in references 7 and 8 are plotted in Fig. (8). The solid data points in this figure represents the results obtained in this investigation and the hollow data points represent those obtained in these references. From this figure it can be seen that the relative changes in the temperature dependence of aluminum alloys as a function of compressive stress can be represented by Eq. 1 with K equal to [illegible] per MPa. The line of best fit of the relative changes in temperature dependence versus tensile stress for aluminum [illegible] is seen to vary slightly from the line of best fit of the other aluminum alloys. This variance is believed to be due to the non-uniform stress induced in the specimens in the case of tensile loading. Therefore, the line of best fit for tensile stress for the aluminum alloys 2024-0, 3003-H251, and 2024-T351 is chosen to represent the true line of best fit of aluminum alloys.

In order to use the temperature dependence method to determine unknown applied stresses, a calibration curve was constructed and is shown in Fig. 9. This calibration curve shows the lines of best fit of the relative percentage changes of the temperature dependence versus applied stress found in Fig. (8). The bands drawn around these lines of best fit are for a 95% confidence level for stress. These bands were determined by means of an inverse regression technique which is described in detail in reference 10. From Figure (9) it can be seen that the relative change in temperature dependence decreases when either compressive or tensile stresses are applied to the specimen. No explanation is available at present for this behavior which makes it difficult to differentiate between tensile and compressive stresses when using the temperature dependence method in the non-destructive evaluation of applied stress in aluminum alloys. To use the calibration curve shown in Fig. (9) to determine applied stresses, the temperature dependence of ultrasonic velocity is measured in the specimen at zero applied stress. This measurement should be repeated several times and the average value is taken as the true value. The specimen is then loaded and the temperature dependence is measured perpendicular to the applied stress. Using the values found above, the relative percentage change in the temperature dependence of ultrasonic velocity if calculated using Eq. 1. This value along with Fig. (9) are used to determine the applied stress. If greater accuracy is required, the temperature dependence at the unknown stress should be measured several times and the average value used to determine the applied stress.

The effects of compressive and tensile stress on the temperature dependence of longitudinal ultrasonic velocity were measured on two specimens

[illegible] ... shows that the relative change of the temperature dependence of ultrasonic velocity is a linear function of the applied stress and can be represented by Eq. [illegible]. The values of [illegible] from Figs. [illegible] are 4.3×10^{-4} per MPa for tensile stresses, -4.5×10^{-4} per MPa for specimens 2 in compression, and -5.63×10^{-4} per MPa for specimen 3 in compression.

In order to use the temperature dependence method to determine unknown applied stress, a calibration curve was constructed and is shown in Fig. 10. From this figure it can be seen that the relative change in temperature dependence increases when either compressive or tensile stress is applied to the specimen. This makes it difficult to differentiate between tensile and compressive stress (as was the case in the perpendicular configuration when using the temperature dependence method in the non-destructive evaluation of applied stress in aluminum [illegible]-T[illegible].

To demonstrate the use of this calibration curve consider an example where the temperature dependence is measured parallel to an applied compressive stress and the relative percentage change in the temperature dependence calculated to be 3.0%. The accuracy of dV/dT measurements in these experiments was found to be ±1.6% using the method of standard error estimation.[11] Thus in this example, the true value of the temperature dependence will be 3±1.6%. The applied stress is read from Fig. (10) as -35 MPa with a lower bound of -57 MPa and an upper bound of -13 MPa. This amount of variance is large and would be unacceptable for many applications. This variance, however, can be reduced by measuring the temperature dependence at the applied compressive stress several times. Consider the case where the temperature dependence at an applied compressive stress is measured five times and the average value is used to calculate a relative percentage change of (dV/dT) of 3.00%. The standard error in this example is $1.6/\sqrt{5} = 0.7\%$. The true value of the relative percentage of (dV/dT) is 3.00 ± .7%. The applied stress is read from Fig. 10 as -35 MPa with a lower bound of -24 MPa and an upper bound of -46 MPa. These examples show that the temperature dependence measurement should be repeated several times when the stress is applied in a direction parallel to the ultrasonic propagation. This will produce a stress measurement with an acceptable amount of error for most applications.

5. ACKNOWLEDGEMENT

This work is supported by the Office of Naval Research under Contract N00014-82-0496.

1. [illegible] and Black, [illegible], Stress and Fatigue in Metals, [illegible] Hall, New York, (1963).

2. [illegible], C. C., J. Metals 31 [illegible], 10, (197[illegible]).

3. Huntington, H. B., Solid State Phys. Rev. 7, 213, (1958).

4. Garber, J. A. and Granato, A. V., Phys. Rev. 131A 3990 (1975).

5. Salama, K., Collins, A. L. W. and Wang, J. J., Proceedings DARPA/AF Review of Progress in Quantitative NDE, p. 265, (1980).

6. Salama, K., Ling, C. K., Proceedings ARPA/AFML Review of Progress in Quantitative NDE, p. 90, (1979).

7. Salama, K., and Ling, C. K., Appl. Phys. 51, 150[illegible] (198[illegible]).

8. Papadakis, E. P., J. Acoust. Soc. Am. 42, 1045, (1967).

9. Salama, K., Ling, C. K., Wang, J. J., Experimental Techniques, [illegible], 14 (1981).

10. Draper, N. R. and Smith, H., Applied Regression Analysis, John Wiley & Sons, Inc., New York, (1981).

11. Lyon, J. A., Dealing with Data, Pergamon Press, Oxford, (1970).

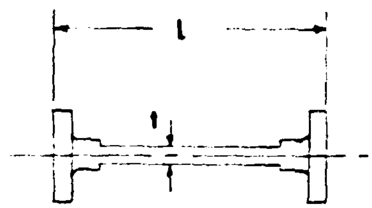


Fig. Specimen used for ultrasonic velocity measurements.

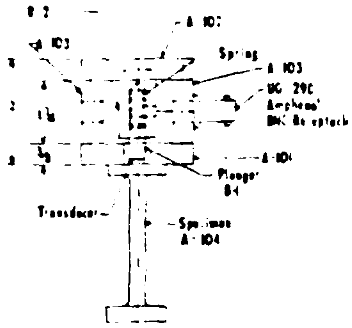


Fig. 2 Transducer holder used for the measurement of ultrasonic velocity when the stress is applied in a direction parallel to ultrasonic propagation.

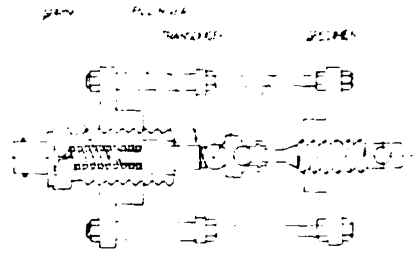


Fig. 3 Transducer holder used for the measurement of ultrasonic velocity when the stress is applied in a direction perpendicular to ultrasonic propagation.

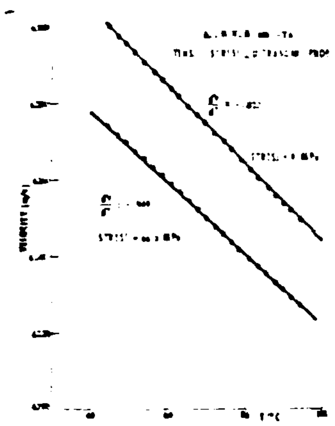


Fig. 4 Effect of applied tensile stress on the temperature dependence of ultrasonic longitudinal velocity in aluminum alloy 6061-T6.

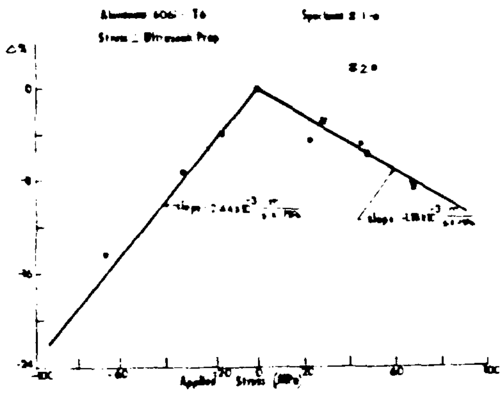


Fig. 5 Relative percentage change in the temperature dependence of ultrasonic longitudinal velocity as a function of applied compressive and tensile stresses in aluminum 6061-T6.

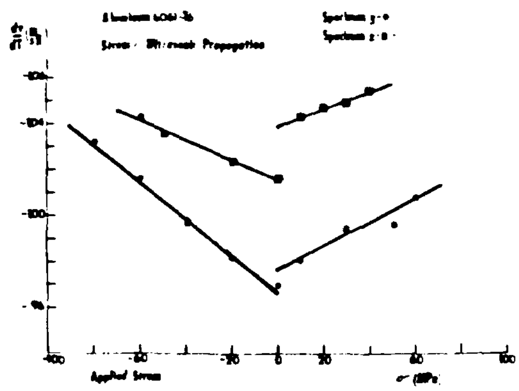


Fig. 6 Effect of applied compressive and tensile stresses on the temperature dependence of ultrasonic longitudinal velocity.

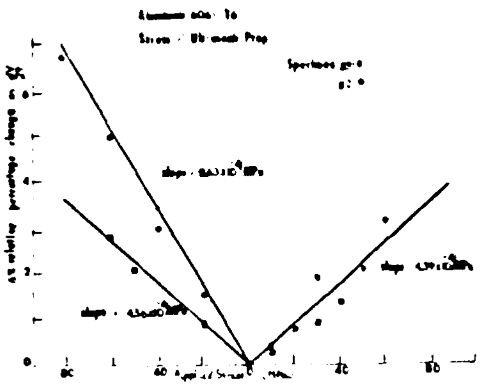


Fig. 7 Relative percentage change in the temperature dependence of ultrasonic longitudinal velocity as a function of applied compressive and tensile stress in aluminum 6061-T6.

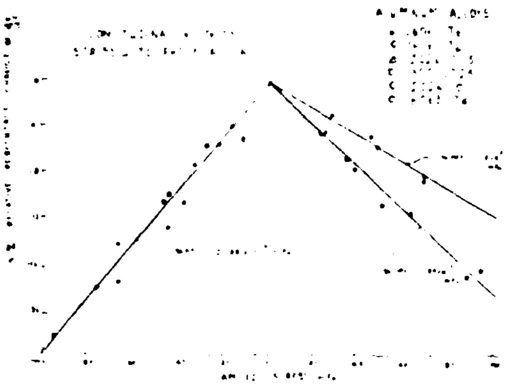


Fig. 8 Relative percentage change in the temperature dependence of longitudinal ultrasonic velocity as a function of applied stress in aluminum alloys. The 'hollow' data points were obtained by Salama, Ling and Wang[9].

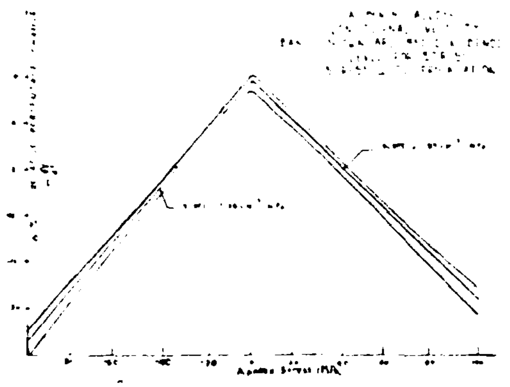


Fig. 9 Calibration curve using the relative change of the temperature dependence of ultrasonic velocity to determine applied stress in aluminum alloys.

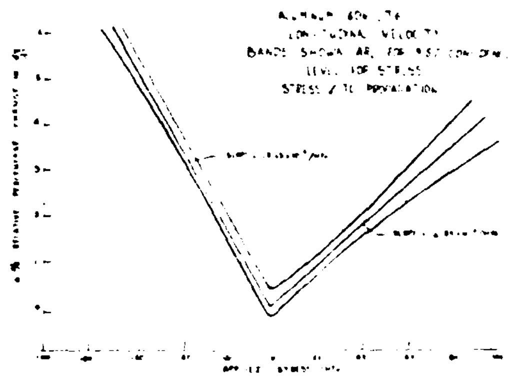


Fig. 10 Calibration curve using the relative change of the temperature dependence of ultrasonic velocity to determine applied stress in aluminum 6061-T6.

APPENDIX II

RELATIONSHIP BETWEEN TEMPERATURE DEPENDENCE
OF ULTRASONIC VELOCITY AND STRESS

Kamel Salama
Department of Mechanical Engineering
University of Houston

ABSTRACT

The temperature dependence of longitudinal ultrasonic velocity in polycrystalline cubic structures has been derived in terms of second-, third- and fourth-order elastic constants. The derivation is made using the Voigt average of elastic constants along with the expressions obtained by Hiki et al. for the temperature dependence of the isothermal second-order elastic constants of single crystals. Comparisons between derived temperature dependence and measured quantities in aluminum and copper, yield values of fourth-order elastic constants which agree with those previously obtained by other investigators using single crystal data. Also, the comparison between the calculated stress derivative of the temperature dependence and the corresponding experimental values, yields reasonable values for the stress dependences of the third- and the fourth-order elastic constants of aluminum and copper. Furthermore, based on the fact that these values are constants for each material, it is concluded that the temperature dependence of ultrasonic velocity in these materials varies linearly with stress as it is observed experimentally.

INTRODUCTION

A great deal of experimental work has been done on the temperature dependence of second-order elastic constants of single- and poly-crystals.[1-7] The data show that the elastic constants are linear functions of temperature up to melting temperatures[8]. Also, recent experiments performed on aluminum[9,10] copper[11] and steel[12-14] elastically deformed, show that the ultrasonic velocity, in the vicinity of room temperature changes linearly with temperature, and the slope of the linear relationship is strongly influenced by the presence of stress. In these materials, the relative changes of the temperature dependence of longitudinal velocity are found to increase by 25% at a stress of 96 MPa in aluminum, by 6% at a stress of 240 MPa in copper, and by a 15% at a stress of 240 MPa in type A533B steel. The results on these materials also indicate that the relative change in the temperature dependence is a linear function of applied stress, and the slope of this linear relationship appears to remain unchanged for specimens of the same material tested. No theoretical explanation, however, is yet available for this linear behavior of the temperature dependence of ultrasonic velocity with stress.

THEORY

Basically the temperature dependences of elastic constants are due to the anharmonic nature of the crystal lattice and are directly related to the higher-order elastic constants[15,16]. Hiki, Thomas and Granato[17] have derived expressions for the temperature dependences of the isothermal second-order elastic constants

using a quasiharmonic anistropic-continuum model. The theory takes account of polarization and orientation dependence of phonon frequencies and their strain derivatives, but assumes the derivatives are wavelength independent. The explicit expanded forms for cubic crystals are given as

$$\left(\frac{\partial C_{11}^{T}}{\partial T}\right)_{\eta} = -K\rho_o \sum_{i=1}^{3N} \{2\gamma_i^{11}\gamma_i^{11} - \frac{1}{2w} [C_{111}^{TT} N_1^2 + C_{112}^{TT}(N_2^2 + N_3^2)$$

$$+ 4C_{111}^{ST} N_1^2 U_1^2 + 4C_{112}^{ST} N_1U_1(N_2U_2+N_3U_3) + 4C_{166}^{ST}[N_1U_1(N_2U_2+N_3U_3) +$$

$$+(N_2^2+N_3^2)U_1^2] + C_{1111}N_1{}^2U_1^2 + 2C_{1112}N_1U_1(N_2U_2+N_3U_3) + C_{1122}(N_2^2U_2^2+N_3^2U_3^2)$$

$$+ 2C_{1123}N_2N_3U_2U_3 + C_{1144}(N_2U_3+N_3U_2)^2 + C_{1155}[(N_1U_3+N_3U_1)^2 +$$

$$(N_1U_2+N_2U_1)^2]]\}, \qquad (1)$$

$$\left(\frac{\partial C_{12}^{T}}{\partial T}\right)_{\eta} = -K\rho_o \sum_{i=1}^{3N} \{2\gamma_i^{11}\gamma_i^{22} - \frac{1}{2w}[C_{112}^{TT}(N_1^2+N_2^2) + C_{123}^{TT}N_3^2 +$$

$$4C_{112}^{ST}N_2U_2(N_1U_1+N_2U_2)$$

$$+4C_{123}^{ST}N_2N_3U_2U_3 + 4C_{144}^{ST}N_3U_2(N_2U_3+N_3U_2) + 4C_{166}^{ST}N_1U_2(N_1U_2+N_2U_1)$$

$$+C_{1112}(N_1^2U_1{}^2+N_2^2U_2{}^2) + 2C_{1122}N_1N_2U_1U_2 + C_{1123}[N_3^2U_3^2+2N_3U_3($$

$$(N_1U_1+N_2U_2)] + C_{1255}[(N_2U_3+N_3U_2)^2+(N_1U_3+N_3U_1)^2]+$$

$$C_{1266}(N_1U_2+N_2U_1)^2]\}, \qquad (2)$$

$$\left(\frac{\partial C_{44}^{T}}{\partial T}\right)_{\eta} = -K\rho_o \sum_{i=1}^{3N} \{2\gamma_i^{23}\gamma_i^{23} - \frac{1}{2w}[C_{144}^{TT}N_1^2 + C_{166}^{TT}(N_2^2 + N_3{}^2) +$$

$$+ 4C_{144}^{ST}N_3N_1U_3U_1 + 4C_{166}^{ST}[(N_2^2+N_3^2)U_3^2+2N_2N_3U_2U_3]+4C_{456}^{ST}(N_1U_3+N_3U_1)N_1U_3$$

$$+C_{1144}N_1^2U_1^2+C_{1155}(N_2^2U_2^2+N_3^2U_3^2)+2C_{1255}(N_1N_2U_1U_2+N_1N_3U_1U_3)$$

$$+2C_{1266}N_2N_3U_2U_3+C_{4444}(N_2U_3+N_3U_2)^2 +$$

$$+C_{4455}[(N_1U_3+N_3U_1)^2+(N_1U_2+N_2U_1)^2]]\}, \tag{3}$$

where the Grüneisen parameters are given as,

$$\gamma_i^{11} = -(1/2w)\{C_{11}^{T}N_1^2+C_{12}^{T}(N_2^2+N_3^2)+2wU_1^2+C_{111}N_1^2U_1^2$$

$$+ C_{144}(N_2U_3+N_3U_2)^2+C_{112}[N_2^2U_2^2+N_3^2U_3^2+2N_1U_1(N_2U_2+N_3U_3)]$$

$$+ 2C_{123}N_2N_3U_2U_3+C_{166}[(N_1U_3+N_3U_1)^2+(N_1U_2+N_2U_1)^2]\}, \tag{4}$$

$$\gamma_i^{23} = -(1/w)\{C_{44}^{T}N_2N_3+wU_2U_3+C_{144}N_1U_1(N_2U_3+N_3U_2)$$

$$+ C_{166}[(N_2^2 + N_3^2)U_2U_3 + N_2N_3(U_2^2 + U_3^2)]$$

$$+ C_{456}[N_2N_3U_1^2 + N_1^2U_2U_3 + N_1U_1(N_2U_3 + N_3U_2)]\} \tag{5}$$

with

$$W = C_{11}^{S}(N_1^2U_1^2+N_2^2U_2^2+N_3^2U_3^2) + C_{44}^{S}[(N_2U_3+N_3U_2)^2 + (N_3U_1+N_1U_3)^2$$

$$+ (N_1U_2+N_2U_1)^2] + 2C_{12}^{S}(N_2N_3U_2U_3+N_3N_1U_3U_1+N_1N_2U_1U_2). \tag{6}$$

In these expressions N and U are, respectively, the propagation and the polarization vectors of the ith normal mode, the superscripts S and I are, respectively, for the adiabatic and the isothermal elastic constants, k is the Boltzmann constant, and ρ_o is the density of the undeformed material. In evaluating the lattice sums of the above equations, Hiki et al.[17] used a 367-point grid over an octant of the Debye sphere. The values of the sums obtained by this method for Cu, Ag, and Au have been shown, however, to be only a few percent larger than those calculated using a pure mode average[18]. Employing the latter method and neglecting the differences between C^{SI}_{ijk} and C^{II}_{ijk}, which have been shown to be small, the temperature dependences of the elastic constants, C_{11}, $C' = \frac{1}{2}(C_{11} - C_{12})$ and C_{44} can be approximately expressed as

$$\frac{\partial C_{11}}{\partial T} = \frac{k\rho_o}{2}\left[\left(\frac{3C_{11} + C_{111}}{C_{11}}\right)^2 - \frac{5C_{111} + C_{1111}}{C_{11}}\right] , \tag{7}$$

$$\frac{\partial C'}{\partial T} = \frac{1}{2}\left(\frac{\partial C_{11}}{\partial T} - \frac{\partial C_{12}}{\partial T}\right) = \frac{k\rho_o}{16C'}(5C_{111} - 5C_{112} + C_{1111} - 4C_{1112} + 3C_{1122}), \tag{8}$$

$$\frac{\partial C_{44}}{\partial T} = \frac{k\rho_o}{4C_{44}}(5C_{166} + C_{144} + 4C_{456} + C_{4444} + C_{4455}). \tag{9}$$

Equation (7) was obtained by considering the longitudinal waves propagating along the [100] direction, while in evaluating the temperature dependences $\partial C'/\partial T$ and $\partial C_{44}/\partial T$, we considered the modes of shear waves propagating along the

[110] direction and polarized parallel to the [110] and [001] directions, respectively.

The expressions for the temperature dependences of the second-order elastic constants show a linear relationship up to the fourth-order elastic constants. Assuming the Cauchy relations[19] hold for the fourth-order elastic constants as,

$$C_{1112} = C_{1155} \quad , C_{1122} = C_{1266} = C_{4444}, \qquad (10)$$
$$C_{1123} = C_{1144} = C_{1255} = C_{1456} = C_{4455}$$

then only four of the fourth-order elastic constants namely, C_{1111}, C_{1112}, C_{1122} and C_{1123} need to be considered. Basically the Cauchy assumption means that the short-range repulsive forces, which contribute the most to the higher-order elastic constants, may be reasonably represented by central forces. Also, assuming that only nearest - neighbor interactions contribute to the fourth-order elastic constants[20], we obtain,

$$C_{1112} = C_{1122} = \frac{1}{2}C_{1111} \quad \text{and} \quad C_{1123} = 0 \qquad (11)$$

and for third-order elastic constants,

$$C_{111} = 2C_{112} = 2C_{166}$$
$$C_{123} = C_{456} = C_{144} = 0$$

This assumption follows from the predominant contribution of the short-range forces to these higher-order elastic contants.

Accordingly eqns. 7, 8, and 9 can respectively be written as

$$\left(\frac{\partial C_{11}}{\partial T}\right) = \frac{-k\varepsilon_o}{2}\left[\left(\frac{3C_{11} + C_{111}}{C_{11}}\right)^2 - \frac{5C_{111} + C_{1111}}{C_{11}}\right] \quad (12)$$

$$\left(\frac{\partial C'}{\partial T}\right) = \frac{k\varepsilon_o}{32C'}\left[5C_{111} + C_{1111}\right] \quad (13)$$

$$\left(\frac{\partial C_{44}}{\partial T}\right) = \frac{k\varepsilon_o}{8C_{44}}\left[5C_{111} + C_{1111}\right] \quad (14)$$

Garbar and Granato[21] used these relationships to determine values of the fourth-order elastic constants of noble metals Cu, Ag and Au. The values of the fourth-order elastic constants obtained were approximately an order of magnitude greater than and opposite in sign from C_{111}, which agrees with the predictions of nearest interactions[22] for a potential of the form $\frac{1}{r^n}$ where $C_{1111}/C_{111} = -\frac{1}{2}(n + 6)$ and a value of 13 was suggested for n.

TEMPERATURE DEPENDENCE OF POLYCRYSTALS

Voigt[23] has shown that the Young's modulus and rigidity modulus of a polycrystal may be calculated from the single-crystal elastic moduli if one assumes that each grain is deformed to the same strain. His results yield,

$$E_V = \frac{(C_{11} - C_{12} + 2C_{44})(C_{11} + 2C_{12})}{(2C_{11} + 3C_{12} + C_{44})} \tag{15}$$

$$G_V = \frac{C_{11} - C_{12} + 3C_{44}}{5} \tag{16}$$

The velocity of longitudinal waves in a polycrystal is related to the Young's modulus and the modulus of rigidity as

$$v_\ell = \frac{G_V}{\rho}\left(\frac{4G_V - E_V}{3G_V - E_V}\right)^{1/2} \tag{17}$$

where ρ is the density of the solid.

Substituting eqns. 15 and 16 into eqn. (17), yields

$$v_\ell = \frac{1}{\sqrt{5\rho}}[5C_{11} - 4C' + 4C_{44}]^{1/2} \tag{18}$$

where C' is $C_{11} - C_{12}/2$.

Considering the density ρ remains unchanged over a short period of temperature and the change in the ultrasonic velocity with temperature is due only to changes in the elastic constants, the temperature dependence of ultrasonic velocity will be given by,

$$\frac{\partial v_\ell}{\partial T} = \frac{1}{2\sqrt{5\rho}}(5C_{11} - 4C' + 4C_{44})^{-1/2}[5\frac{\partial C_{11}}{\partial T} - 4\frac{\partial C'}{\partial T} + 4\frac{\partial C_{44}}{\partial T}] \tag{19}$$

Substituting for the temperature dependences $\frac{\partial C_{11}}{\partial T}$, $\frac{\partial C'}{\partial T}$ and $\frac{\partial C_{44}}{\partial T}$ from eqns. (12, 13 and 14), eqn. (19) takes the form,

$$\frac{\partial v_\ell}{\partial T} = \frac{k\varepsilon_o}{\sqrt{5\rho_o}}(5C_{11} - 4C' + 4C_{44})^{-1/2}\Big[-\frac{45}{4} - \frac{5C_{111}}{4C_{11}} - \frac{5C_{111}^{\,2}}{4C_{11}^{\,2}} + \frac{5C_{1111}}{4C_{11}}$$

$$- \frac{5C_{111}}{16C'} - \frac{C_{1111}}{16C'} + \frac{5C_{111}}{4C_{44}} + \frac{C_{1111}}{4C_{44}} \qquad (20)$$

Equation (20) represents the temperature dependence of longitudinal velocity in a polycrystalline solid in terms of the second-, third- and fourth-order elastic constants of the single crystal structure of that solid. A similar relationship, however more complicated, may also be obtained using the Reuss[24] averaging procedure in calculating the Young's and Rigidity moduli, which assumes each grain is subject to constant stress.

CHANGE OF TEMPERATURE DEPENDENCE OF ULTRASONIC VELOCITY WITH STRESS

1. Aluminum

Measurements of the ultrasonic longitudinal velocity in the temperature range between 180 and 260 K on the aluminum alloys 2024-0 and 6063-T4, have shown that the velocity decreases linearly with temperature, and the value of the temperature dependence dV_{ℓ}/dT is in the range between -92.5 and -111.1 cm/s.K. Similar results were also obtained on the aluminum alloys 2024-T351, 5005-T251, and 6061-T4, where the values of dV_{ℓ}/dT are respectively found to be -118.7, -125.1 and -130.4 cm/s.K. The average value of the temperature dependence of longitudinal velocity in polycrystalline aluminum, -115 m/s.K, and the values of the second- and third-order elastic constants obtained by Thomas[25], C_{11} = 1.0675, C' = 0.257, C_{44} = 0.2834 and C_{111} = -10.76 in units of 10^{12}dyne/cm^2, are substituted in to eqn. (20). The value of the fourth-order elastic constant C_{1111} is found to be ±86.5 x 10^{12}dyne/cm^2, which is opposite in sign, and about a factor of 8 larger than that of measured C_{111}. This agrees with the predictions of nearest-neighbor interactions for a potential of the form $\frac{1}{r^n}$ with n = 10. It also agrees with the findings of Garbor and Granato in their determination of the fourth-order elastic constants of the noble metals Cu, Ag and Au.

Equation (20) indicates that the temperature dependence of ultrasonic velocity is a constant quantity which may be computed knowing the values of the second-, third- and fourth-order elastic constants. Again considering the density remains unchanged as a function of stress, the derivative of the right-hand side of eqn. (20) with respect to stress will be a function of the second-,

third- and fourth-order elastic constants, and their derivatives with respect to stress. The derivative of the left-hand side of this equation with respect to stress, $\frac{\partial}{\partial \sigma}\left(\frac{\partial V_{\ell}}{\partial T}\right)$ will then be a constant as long as the derivatives of the elastic constants with respect to stress are constants. Recent experiments on aluminum, copper and steel, have shown that the temperature dependence of ultrasonic velocity in these materials is a linear function of applied stress. The slope of this linear relationship, however, is to be different in different materials, and and differs according to the relative direction of stress with respect to the directions of propagation and polarization.

In order to obtain quantitative determination of the derivative $\frac{\partial}{\partial \sigma}\left(\frac{\partial V_{\ell}}{\partial T}\right)$, a computer program was developed to compute the value of the quantity $\partial^2 V_{\ell}/\partial\sigma\partial T$ for a given values of the second-, third- and fourth-order elastic constants and their derivatives with respect to stress. The program is based on Newton's backward difference formula for polynomial approximation[26], which does not give the exact differential but rather an approximate differentiation of the function. The accuracy of the program was tested for functions of known derivatives, and the results of the derivatives at given values of the function were found to be equal to those of the exact solution within 0.1%.

The second- and third-order elastic constants of aluminum single crystals obtained by Thomas along with the value of the fourth-order elastic constants $\pm 86.5 \times 10^{12}$ dyne/cm^2 obtained from the temperature dependence calcuations are used to evaluate $\frac{\partial^2 V_{\ell}}{\partial \sigma \partial T}$. Also the values of the stress dependences of the

second-order elastic constants $\frac{\partial C_{11}}{\partial \sigma}$, $\frac{\partial C'}{\partial \sigma}$ and $\frac{\partial C_{44}}{\partial \sigma}$ for stress applied perpendicular to propagation, obtained by Thomas[25] are used in the computer program to calculate $\frac{\partial^2 v_\ell}{\partial \sigma \partial l}$. These values for stress applied in tension are +1.67, 0.61 and -2.46 respectively. The value -0.61 is the average of -0.32 and -0.90 obtained with stress applied in two different directions. For stress dependences of the third- and fourth-order elastic constants $\frac{\partial C_{111}}{\partial \sigma}$ and $\frac{\partial C_{1111}}{\partial \sigma}$, no values are available in the literature, and accordingly some estimates were made. Using $\frac{\partial C_{111}}{\partial \sigma} = 10$ and $\frac{\partial C_{1111}}{\partial \sigma} = 100$ is found to yield a value of 0.228×10^{-7} $\frac{cm/s.k}{dyne/cm^2}$ for $\frac{\partial^2 v_\ell}{\partial \sigma \partial l}$ which is in excellent agreement with the value 0.242×10^{-7} $cm/s.k/dyne/cm^2$ obtained experimentally[11]. The values of $\frac{\partial C_{111}}{\partial \sigma}$ and $\frac{\partial C_{1111}}{\partial \sigma}$ used to yield the agreement between calculations and experiment are of the same order of magnitude to be expected for these quantities. The values of the derivative $\frac{\partial^2 v_\ell}{\partial \sigma \partial l} = 0.12 \times 10^{-7} \frac{cm/s.k}{dyne/cm^2}$ when both $\frac{\partial C_{111}}{\partial \sigma}$ and $\frac{\partial C_{1111}}{\partial \sigma}$ are equal to zero. Also changing the values of $\frac{\partial C_{111}}{\partial \sigma}$ and $\frac{\partial C_{1111}}{\partial \sigma}$ by ±50% changes the value of the derivative by ±25%. Table I contains the values of $\frac{\partial^2 v_\ell}{\partial \sigma \partial \sigma}$ at different values of $\frac{\partial C_{111}}{\partial \sigma}$ and $\frac{\partial C_{1111}}{\partial \sigma}$.

Table 1. Values of the derivatives $\frac{\partial}{\partial\sigma}\left(\frac{\partial V_\lambda}{\partial T}\right)$ in aluminum at various values of stress dependences $-\frac{\partial C_{111}}{\partial\sigma}$ and $\frac{\partial C_{1111}}{\partial\sigma}$. Calculations are made using $C_{11} = 1.068$, $C' - 0.2517$, $C_{44} = 0.2854$, $C_{111} = -10.76$ and $C_{1111} = 86.5$ in units of 10^{12} dynes/cm^2, $\frac{\partial C_{11}}{\partial\sigma} = 1.67$, $\frac{\partial C'}{\partial\sigma} = -0.61$, $\frac{\partial C_{44}}{\partial\sigma} = -2.46$.

$\frac{\partial C_{111}}{\partial\sigma}$	$\frac{\partial C_{1111}}{\partial\sigma}$	$\frac{\partial}{\partial\sigma}\left(\frac{\partial V_\lambda}{\partial T}\right)$ $\frac{\text{cm/s.K}}{\text{dynes/cm}^2}$
0	0	$0.116 \times 10^{-}$[illegible]
5	50	$0.172 \times 10^{-}$[illegible]
10	100	$0.228 \times 10^{-}$[illegible]
15	150	$0.284 \times 10^{-}$[illegible]
Experiment		0.242×10^{-7}

2. Copper

The second- and the third-order elastic constants of single crystal copper have been measured by Hiki and Granato[20]. The values obtained are $C_{11} = 1.661$, $C' = 0.231$, $C_{44} = 0.756$ and $C_{111} = -12.71$ in units of 10^{12} dynes/cm^2. Also the temperature dependence of ultrasonic longitudinal velocity $\partial v_\ell/\partial T$ in CDA 110 polycrystalline copper has been measured by Salama and Ling[11]. The results obtained on annealed specimens averaged a value of -0.4865 m/s.K for $(\partial v_\ell/\partial T)$, while those on as received specimens have an average of -0.521 m/s.K. Substituting the average of these two quantities -0.504 m/s.K for $\partial v_\ell/\partial T$ in eqn. (20), and using the values of the second- and third-order elastic constants obtained by Hiki and Granato[20], we obtain a value of $C_{1111} = 93.55 \times 10^{12}$ dyne/cm^2. This value is opposite in sign and about a factor of 7.5 larger than that of the third-order elastic constant C_{111}. It also lies within the range 87.7 - 107 x 10^{12} dyne/cm^2 obtained by *Garbar and Granato*[21] from their analysis of the temperature dependence of the second-order elastic constants of single crystals of copper.

The values of the stress dependence of the second-order elastic constants C_{11}, C' and C_{44} in copper were calculated using the measurements of Hiki and Granato[20], and found to be $dC_{11}/d\sigma = -0.66$, $dC'/d\sigma = -4.17$ and $dC_{44}/d\sigma = 1.41$ for tensile stress. Each of the values of $dC'/d\sigma$ and $dC_{44}/d\sigma$ are the average of the values obtained on three different combinations of stress, propagation and polarization directions. Each of these two quantities depend strongly on the relative orientation of the direction in which the uniaxial stress is applied with respect to the polarization direction. Using these values along with those of

the second-, third- and fourth-order elastic constants of copper in the computer program to obtain the derivative $\frac{\partial}{\partial\sigma}\left(\frac{\partial V_\ell}{\partial T}\right)$, a good agreement between theory and experiment for this quantity is obtained with $\partial C_{111}/\partial\sigma = 8$ and $\partial C_{1111}/\partial\sigma = 80$. The calculated value of $\frac{\partial^2 V_\ell}{\partial\sigma\partial T}$ is found to be 0.129×10^{-8} cm/s.K/dyne/cm^2, while that obtained from measurements of the temperature dependence of ultrasonic velocity as a function of applied stress is equal to 0.125×10^{-8} cm/s.K/dyne/cm^2. These values are a factor of 20 smaller than those obtained in aluminum. Nevertheless, the values of the stress dependences of the third- and fourth-order elastic constants $\frac{\partial C_{111}}{\partial\sigma} = 8$ and $\frac{\partial C_{1111}}{\partial\sigma} = 80$ obtained in copper are of the same magnitude as of 10 and 100 computed for $\frac{\partial C_{111}}{\partial\sigma}$ and $\frac{\partial C_{1111}}{\partial\sigma}$ respectively in aluminum. Contrary to aluminum, however, the calculated value of the slope of the relationship between the temperature dependence of the longitudinal velocity $\frac{\partial V_\ell}{\partial T}$ and applied tensile stress σ, is found to be sensitive to the values used for $\frac{\partial C_{111}}{\partial\sigma}$ and $\frac{\partial C_{1111}}{\partial\sigma}$. Table II lists the calculated values of $\frac{\partial^2 V_\ell}{\partial\sigma\partial T}$ in copper using different values of $\frac{\partial C_{111}}{\partial\sigma}$ and $\frac{\partial C_{1111}}{\partial\sigma}$ along with the same values of the second-, third- and fourth-order elastic constants and the stress derivatives of the second-order elastic constants.

Table II. Values of the derivative $\frac{\partial}{\partial\sigma}\left(\frac{\partial v_k}{\partial T}\right)$ in copper at various values of the stress dependences $\frac{\partial C_{111}}{\partial\sigma}$ and $\frac{\partial C_{1111}}{\partial\sigma}$. Calculations are made using $C_{11} = 1.661$, $C' = 0.231$, $C_{44} = 0.756$, $C_{111} = -12.71$ and $C_{1111} = 93.55$ in units of 10^{12}dyne/cm^2. $\frac{\partial C_{11}}{\partial\sigma} = -0.66$, $\frac{\partial C'}{\partial\sigma} = -1.17$ and $\frac{\partial C_{44}}{\partial\sigma} = 1.41$.

$\frac{\partial C_{111}}{\partial\sigma}$	$\frac{\partial C_{1111}}{\partial\sigma}$	$\frac{\partial}{\partial\sigma}\left(\frac{\partial v_k}{\partial T}\right)$ $\frac{\text{cm/s.K}}{\text{dynes/cm}^2}$
0	0	-0.161×10^{-8}
8	80	$+0.129 \times 10^{-8}$
7	70	$+0.093 \times 10^{-8}$
9	90	$+0.166 \times 10^{-8}$
9	70	$+0.135 \times 10^{-8}$
7	90	$+0.124 \times 10^{-8}$
Experiment		$+0.125 \times 10^{-8}$

ACKNOWLEDGEMENT

The author would like to thank Drs. J. H. Cantrell and J. S. Heyman from NASA Langley Research Center for very valuable discussions. He also would like to acknowledgements the financial support from the Office of Naval Research under Contract N00014-82-K-0496.

REFERENCES

1. J. Lesse and A. E. Lord, Jr., J. Appl. Phys. 39, 3986 (1968).
2. N. Soga, J. Appl. Phys. 37, 3416 (1966).
3. J. Vallin, M. Mongy, K. Salama and O. Beckman, J. Appl. Phys. 35, 1825 (1964).
4. P. L. Young and A. Bienenstock, J. Appl. Phys. 42, 3008 (1971).
5. D. Gerlich and E. S. Fisher, J. Phys. Chem. Solids 30, 1197 (1969).
6. Y. A. Chang and L. Himmel, J. Appl. Phys. 37, 3567 (1966).
7. J. M. Dickinson and P. E. Armstrong, J. Appl. Phys. 38, 602, (1967).
8. R. Lowrie and A. E. Gonas, J. Appl. Phys. 38, 4505 (1967).
9. K. Salama and C. K. Ling, Proc. ARPA/AFML Review of Progress in Quantitative NDE, p. 96, (1979).
10. K. Salama, A. L. W. Collins and J. J. Wang, Proc. DARPA/AF Review of Progress in Quantitative NDE, p. 265 (1980).
11. K. Salama and C. K. Ling, J. Appl. Phys. 51, 1505 (1980).
12. K. Salama, C. K. Ling and J. J. Wang, Experimental Techniques 5, 14 (1981).
13. K. Salama and J. J. Wang, Proc. German-United States Workshop on Research and Development to New Procedures in NDT, Ed. P. Höller, Springer-Verlag, p. 539 (1982).
14. K. Salama, G. C. Barber and N. Chandrasekaran, Proc. IEEE Ultrasonic Ultrasonic Symposium, p. 877 (1982).

15. H. B. Huntington, Solid State Physics 7, 213 (1958).

16. K. Salama and G. A. Alers, Phys. Rev. 161, 673 (1967).

17. Y. Hiki, J. F. Thomas, Jr. and A. V. Granato, Phys. Rev. 153, 764 (1967).

18. J. F. Thomas, Jr. (Private Communication).

19. F. Birch, Phys. Rev. 71, 809 (1947).

20. Y. Hiki, and A. V. Granato, Phys. Rev. 144, 411 (1966).

21. J. A. Garber and A. V. Granato, Phys. Rev. 11B, 3998 (1975).

22. J. B. Gibson, A. N. Goland, M. Milgram and G. A. Vineyard, Phys. Rev. 120, 1229 (1968).

23. W. Voigt, Lehrbuch der Kristallphysik, Teubner Leipzig, p. 962, (1910).

24. A. Reuss, Z. Angew, Math. Mech. 9, 49 (1929).

25. J. F. Thomas, Jr., Phys. Rev. 175, 955 (1968).

LMED

8